Complexity of the resolution of univariate polynomial equations over finite discrete dynamical systems y of the resolution of
olynomial equations
discrete dynamical
systems
_{seminarl3s}
_{Thursday, November 14th}

Seminar I3S

Marius ROLLAND, CNRS & LIS, Marseille

A LITTLE CONTEXT

What is a finite discrete dynamical system (FDDS) ?

What is a finite discrete dynamical system (FDDS) ?

What is a finite discrete dynamical system (FDDS) ?

Operation : Sum

Property of product

connected FDDS A, B with cycle lengths a, b I **Property of product**
The product of two connected FDDS A , B with cycle lengths a , b has
 $gcd(a, b)$ connected components with cycle length $lcm(a, b)$. **Property of product**
ct of two connected FDDS A , B with cycle lengths a , b has
connected components with cycle length $lcm(a, b)$.

The Semiring $(D, +, \times)$

The set **D** of FDDS up to isomorphism with the **alternative execution** as addition and **the Shifter Semiring (D, +,** \times **)**
The set **D** of FDDS up to isomorphism with
the **alternative execution** as addition and the
synchronus execution as multiplication is a
commutative semiring. commutative semiring.

Polynomial equations over FDDS

• Undecidable in the general case : $P_1(\vec{X}) = P_2(\vec{X})$

Polynomial equations over FDDS **Polynomial equations over FDDS**
• Undecidable in the general case : $P_1(\vec{X}) = P_2(\vec{X})$
• NPC in general for the equation $P(\vec{X}) = B$ **Polynomial equations over FDI**
• Undecidable in the general case : $P_1(\vec{X})$ =
• NPC in general for the equation $P(\vec{X}) = B$

-
-

Polynomial equations over FDDS **Polynomial equations over FDDS**
• Undecidable in the general case : $P_1(\vec{X}) = P_2(\vec{X})$
• NPC in general for the equation $P(\vec{X}) = B$

-
-
- *NPC* in general for the equation $P(\vec{X}) = B$
• In P for the monomial univariate equations
(*A X^k* = *B*) with connected result. **Polynomial equations over FDDS**
• Undecidable in the general case : $P_1(\vec{X}) = P_2(\vec{X})$
• NPC in general for the equation $P(\vec{X}) = B$
• In P for the monomial univariate equations
($A X^k = B$) with connected result. Undecidable in the general case : $P_1(\vec{X}) = P_2(\vec{X})$
 NPC in general for the equation $P(\vec{X}) = B$

In P for the monomial univariate equations

(*A X^k* = *B*) with connected result.

OUR NEW RESULTS

1. The behavior of transient nodes

First tool : Unroll

**First tool : Unroll
Main idea :** Transform a FDDS into a forest where each
tree is rooted in a periodic nodes and represents all
possible paths in the FDDS between a node and its root. First tool : Unroll
Main idea : Transform a FDDS into a forest where each
tree is rooted in a periodic nodes and represents all
possible paths in the FDDS between a node and its root. possible paths in the FDDS between a node and its root.

Tree product

Tree product

Tree product

The Semiring $(\mathsf{U}(\mathsf{D}), +, \blacksquare)$

The set $U(D)$ of FDDS up to isomorphism The Semiring $(U(D), +, \blacksquare)$
The set U(D) of FDDS up to isomorphism
with the disjoint union as addition and the
tree product as multiplication is a **The Semiring** $(U(D), +, \blacksquare)$
The set U(D) of FDDS **up to isomorphism**
with the **disjoint union** as addition and the
tree product as multiplication is a
commutative semiring. commutative semiring.

Result over Unroll:
The between **D** and U(**D**) ⇒

• *U* is a morphisme between **D** and $U(D) \Rightarrow$
 $U(\sum_{i=1}^{n} A_i X^i) = \sum_{i=1}^{n} U(A_i) U(X)^i$.

Result over Unroll:
The between **D** and U(**D**) ⇒

- *U* is a morphisme between **D** and $U(D) =$
 $U(\sum_{i=1}^{n} A_i X^i) = \sum_{i=1}^{n} U(A_i) U(X)^i$. **For all Allen Island Set (Allen Islam Burn I)**:

For all unroll tree A, B and $U(D) \Rightarrow$

for all unroll tree A, B and T.
- **U** is a morphisme between **D** and $U(D) \Rightarrow$
 $U(\sum_{i=1}^{n} A_i X^i) = \sum_{i=1}^{n} U(A_i) U(X)^i$.

 There exists an order such that $A \leq B \Leftrightarrow AT \leq B$ T for all unroll tree A, B and T.

Second tool : Cut

Second tool : Cut
Main idea : in a forest, consider only the nodes whose
depth is less than a certain *n*. **Second tool : Cut**
Main idea : in a forest, consider only the nod
depth is less than a certain *n*.

Second tool : Cut

Result over Cut:
FDDS A, B we have that $C_n(AB) =$ • For all integer n and FDDS A , B we have that $C_n(AB) = C_n(A)C_n(B)$. .

- Result over Cut:
FDDS A, B we have that $C_n(AB) =$ **•** For all integer *n* and FDDS *A*, *B* we have that $C_n(AB) = C_n(A)C_n(B)$.
• There exists an integer *n* such that .
- **•** For all integer n and FDDS A, B we have that $C_n(A)C_n(B)$.

 There exists an integer n such that $U(\sum_{i=1}^m A_i X^i) = U(B) \Leftrightarrow C_n \left(U(\sum_{i=1}^m A_i X^i)\right) = C_n(U(B)).$ **for all integer** *n* **and FDDS A, B we have that** $C_n(AB) = C_n(A)C_n(B)$ **.**
 • There exists an integer *n* **such that** $U(\sum_{i=1}^m A_i X^i) = U(B) \Leftrightarrow C_n \left(U(\sum_{i=1}^m A_i X^i)\right) = C_n(U(B)).$

- Result over Cut:
FDDS A, B we have that $C_n(AB) =$ **•** For all integer *n* and FDDS *A*, *B* we have that $C_n(AB) = C_n(A)C_n(B)$.
• There exists an integer *n* such that .
- **•** For all integer n and FDDS A, B we have that $C_n(AB) = C_n(A)C_n(B)$.

 There exists an integer n such that
 $U(\sum_{i=1}^m A_i X^i) = U(B) \Leftrightarrow C_n \left(U(\sum_{i=1}^m A_i X^i) \right) = C_n(U(B))$.

 The previous order is such that $C_n(A) \leq C_n(B) \Leftrightarrow C_n(A$ **for all integer** *n* **and FDDS** *A***,** *B* **we have that** $C_n(AB) =$ **
** $C_n(A)C_n(B)$ **.

• There exists an integer** *n* **such that
** $U(\sum_{i=1}^m A_i X^i) = U(B) \Leftrightarrow C_n(U(\sum_{i=1}^m A_i X^i)) = C_n(U(B))$ **.

• The previous order is such that C_n(A) \leq C_n(B) \Leftrightarrow**
-

Form of forests
e a polynomial over U_n and $X = \sum_{i=1}^k U_i$ Form of forests

Let $P = \sum_{i=1}^{m} A_i X^i$ be a polynomial over U_n and $X = \sum_{i=1}^{m} A_i X^i$ be a polynomial over U_n and $X = \sum_{i=1}^{m} A_i X^i$

1. The tree min $(P(X))$ is isomorphic to $a_1 x_1$ ⁱ Form of forests

Let $P = \sum_{i=1}^{m} A_i X^i$ be a polynomial over U_n and $X = \sum_{i=1}^{k} x_i$

with $x_i \leq x_{i+1}$. Then, there exists $i \in \{1, ..., m\}$ such that :

1. The tree min $(P(X))$ is isomorphic to $a_1 x_1^i$

2. min $(P(X) - P(\sum_{i$ Form of forests
 $=\sum_{i=1}^{m} A_i X^i$ be a polynomial over U_n and $X = \sum_{i=1}^{k} x_i$
 $x_i \leq x_{i+1}$. Then, there exists $i \in \{1, ..., m\}$ such that :

1. The tree min $(P(X))$ is isomorphic to $a_1x_1^i$

2. min $(P(X) - P(\sum_{j=1}^{k-1} x_j))$ Form of forests
 $=\sum_{i=1}^{m} A_i X^i$ be a polynomial over U_n and $X = \sum_{i=1}^{k} x_i$
 $x_i \le x_{i+1}$. Then, there exists $i \in \{1, ..., m\}$ such that :

1. The tree min $(P(X))$ is isomorphic to $a_1 x_1$ ^{*i*}

2. min $(P(X) - P(\sum_{j=1}^{k-1} x$

-
- $a_1x_1^{i-1}x_k$ for all $k \geq 2$

Where $a_1 = \min(A_i)$.

Proof of the assertion **Proof of the assertior**
• Let *i* such that $min(P(X)) \in A_i X^i$. Proof of the assertior
 $\min(P(X)) \in A_i X^i$. **Proof of the assertion**
• Let *i* such that $\min(P(X)) \in A_i X^i$.

Proof of the asset
• Let *i* such that $min(P(X)) \in A_i X^i$. *i*. • Let *i* such that $min(P(X)) \in A_i X^i$.

Proof of the assertion Proof of the assertion
 $\min(P(X)) \in A_i X^i$.

we that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i$

- *i*.
- Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P)$ • Let *i* such that $min(P(X)) \in A_i X^i$.
• For all *j*, we have that $min(X^j) = x_1^j \Rightarrow min(P(X)) \in A$ $\lim_{x \to \infty} (p(x))$ $i^{\mathcal{X}}1$. i and i a . • Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i x_1^i$. • Let *i* such that $\min(P(X)) \in A_i X^i$.

• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i$.
 $\sum_{i=1}^{n} P(X^i)$

Proof of the assertion Proof of the assertion
 $\min(P(X)) \in A_i X^i$.

we that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$

- *i*.
- Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P$
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow n$ • Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A$
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$ $\lim_{x \to \infty} (p(x))$ $i^{\mathcal{X}}1$. i and i a . • Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i x_1^i$.
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X)) = a_j$. • Let *i* such that $\min(P(X)) \in A_i X^i$.

• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i$.

• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$.
- For all $a_j \in A_i$, we have that $a_1x_1{}^i \leq a_jx_1{}^i \Rightarrow a_jx_1{}^i$ ϵ_j $x_1 \rightarrow$ min($P(x)$) – c $i \Rightarrow min(P()$ $1^{\lambda}1$. *i*.

Proof of the assertion Proof of the assertion
 $\min(P(X)) \in A_i X^i$.

we that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$

have that :
 $a_1 x_1^i \le \min(A_j) x_1^j$

- *i*.
- Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P$
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow n$ • Let *i* such that $\min(P(X)) \in A_i X^i$.
• Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A$.
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$.
• For all *j*, *k*, we have that : $\lim_{x \to \infty} (p(x))$ $i^{\mathcal{X}}1$. i and i a . • Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i x_1^i$.
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X)) = a_i$.
• For all *j*, *k*, we have that :
 $a_1 x_1^i \le \min(A_j) x_1^j$ • Let *i* such that $\min(P(X)) \in A_i X^i$.

• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i$.

• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$

• For all *j*, *k*, we have that :
 $a_1 x_1^i \le \min(A_j) x_1^j$
- For all $a_j \in A_i$, we have that $a_1x_1{}^i \leq a_jx_1{}^i \Rightarrow a_jx_1{}^i$ ϵ_j $x_1 \rightarrow$ min($P(x)$) – c $i \Rightarrow min(P()$ $1^{\lambda}1$. *i*.
-

$$
a_1x_1^i \leq \min(A_j)x_1^j
$$
Proof of the assertion

- *i*.
- Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P$
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow n$ • Let *i* such that $\min(P(X)) \in A_i X^i$.
• Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A$.
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$.
• For all *j*, *k*, we have that : $\lim_{x \to \infty} (p(x))$ $i^{\mathcal{X}}1$. i and i a . Proof of the assertion
 $\min(P(X)) \in A_i X^i$.

we that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$

have that :
 $a_1 x_1^i \le \min(A_j) x_1^j$ • Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i x_1^i$.
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X)) = a_1 x_1^i$.
• For all *j*, *k*, we have that :
 $a_1 x_1^i \le \min(A_j) x_1^$
- For all $a_j \in A_i$, we have that $a_1x_1{}^i \leq a_jx_1{}^i \Rightarrow a_jx_1{}^i$ ϵ_j $x_1 \rightarrow$ min($P(x)$) – c $i \Rightarrow min(P()$ $1^{\lambda}1$. *i*.
-

\n- Let *i* such that
$$
\min(P(X)) \in A_i X^i
$$
.
\n- For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i x_1$.
\n- For all $a_j \in A_i$, we have that $a_1 x_1^i \leq a_j x_1^i \Rightarrow \min(P(X)) =$.
\n- For all *j*, *k*, we have that:\n
	\n- $a_1 x_1^i \leq \min(A_j) x_1^j$
	\n- $a_1 x_1^{i-1} \leq \min(A_j) x_1^{j-1}$
	\n\n
\n

Proof of the assertion

- *i*.
- Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P$
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow n$ • Let *i* such that $\min(P(X)) \in A_i X^i$.
• Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A$.
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$.
• For all *j*, *k*, we have that : $\lim_{x \to \infty} (p(x))$ $i^{\mathcal{X}}1$. i and i a . Proof of the assertion
 $\min(P(X)) \in A_i X^i$.

we that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$

have that :
 $a_1 x_1^i \le \min(A_j) x_1^j$
- For all $a_j \in A_i$, we have that $a_1x_1{}^i \leq a_jx_1{}^i \Rightarrow a_jx_1{}^i$ ϵ_j $x_1 \rightarrow$ min($P(x)$) – c $i \Rightarrow min(P()$ $1^{\lambda}1$. *i*.
-

Proof of the assertion
\n• Let *i* such that
$$
\min(P(X)) \in A_i X^i
$$
.
\n• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i x_1^i$.
\n• For all *a_j* ∈ *A_i*, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X)) = c$
\n• For all *j*, *k*, we have that:
\n
$$
a_1 x_1^i \le \min(A_j) x_1^j
$$
\n
$$
\Leftrightarrow a_1 x_1^{i-1} \le \min(A_j) x_1^{j-1}
$$
\n
$$
\Leftrightarrow a_1 x_1^{i-1} x_k \le \min(A_j) x_1^{j-1} x_k.
$$

Proof of the assertion

- *i*.
- Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P$
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow n$ • Let *i* such that $\min(P(X)) \in A_i X^i$.
• Let *i* such that $\min(P(X)) \in A_i X^i$.
• For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A$.
• For all $a_j \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$.
• For all *j*, *k*, we have that : $\lim_{x \to \infty} (p(x))$ $i^{\mathcal{X}}1$. i and i a . Proof of the assertion
 $\min(P(X)) \in A_i X^i$.

we that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i$, we have that $a_1 x_1^i \le a_j x_1^i \Rightarrow \min(P(X))$

have that :
 $a_1 x_1^i \le \min(A_j) x_1^j$
- For all $a_j \in A_i$, we have that $a_1x_1{}^i \leq a_jx_1{}^i \Rightarrow a_jx_1{}^i$ ϵ_j $x_1 \rightarrow$ min($P(x)$) – c $i \Rightarrow min(P()$ $1^{\lambda}1$. *i*.
-

\n- \n**Proof of the assertion**\n
\n- \n Let *i* such that
$$
\min(P(X)) \in A_i X^i
$$
.\n
\n- \n For all *j*, we have that $\min(X^j) = x_1^j \Rightarrow \min(P(X)) \in A_i x_1^i$.\n
\n- \n For all *a_j* ∈ *A_i*, we have that $a_1 x_1^i \leq a_j x_1^i \Rightarrow \min(P(X)) = c$.\n
\n- \n For all *j*, *k*, we have that:\n
$$
a_1 x_1^{i-1} \leq \min(A_j) x_1^{j-1}
$$
\n
$$
\Leftrightarrow a_1 x_1^{i-1} x_k \leq \min(A_j) x_1^{j-1} x_k.
$$
\n
\n- \n For all *j*, we have that $\min(X^{j-1} x_k) = \min(X^{j-1}) x_k = x$.\n
\n- \n So, $\min(A_j) x_1^{j-1} x_k \leq \min(A_j) y x_k \leq a y x_k$, for all $y \in A_j$.\n
\n

- $j-1$) $x_k = x_1^{j-1}x_k$
- So, $\min(A_j)x_1^{j-1}x_k \leq \min(A_j)yx_k \leq a y x_k$, for all $y \in X^{j-1}$, $a \in A_j$. .
39

For all positive integer n :

1. All positive integer n **:**
1. All polynomial functions of U_n are injective (simple induction)

- **1.** All positive integer n :

1. All polynomial functions of U_n are injective (simple induction)

2. we can solve in polynomial time $P(X) = B$ over U_n . **2. We can solve in the control CONSEQUENCES**
2. We can solve in polynomial time $P(X) = B$ over U_n .
2. we can solve in polynomial time $P(X) = B$ over U_n .
-

- **1.** All positive integer n :

1. All polynomial functions of U_n are injective (simple induction)

2. we can solve in polynomial time $P(X) = B$ over U_n .

1. Taking a coefficient i , **2. We can solve in polynomial functions of** U_n **are injective (simple induction 2.** we can solve in polynomial time $P(X) = B$ over U_n .
1. Taking a coefficient *i*,
- -

- **1.** All positive integer n :

1. All polynomial functions of U_n are injective (simple induction)

2. we can solve in polynomial time $P(X) = B$ over U_n .

1. Taking a coefficient i , **2.** We can solve in teger n in the solve in polynomial functions of U_n are injective (simple induction 2. we can solve in polynomial time $P(X) = B$ over U_n .
1. Taking a coefficient *i*,
2. Compute $x = \sqrt{\frac{\min(B)}{\min(A_i)}},$
- -

Consequence
l positive integer *n*:
All polynomial functions of
$$
U_n
$$
 are in
we can solve in polynomial time $P($.
1. Taking a coefficient *i*,
2. Compute $x = \sqrt[i]{\frac{\min(B)}{\min(A_i)}}$,

- **1.** All positive integer n :

1. All polynomial functions of U_n are injective (simple induction)

2. we can solve in polynomial time $P(X) = B$ over U_n .

1. Taking a coefficient i ,
- -

2. we can solve in polynomial time over . 1. Taking a coefficient , 2. Compute ⁼ , 3. Add to the tree షభ until ,

- **1.** All positive integer n :

1. All polynomial functions of U_n are injective (simple induction)

2. we can solve in polynomial time $P(X) = B$ over U_n .

1. Taking a coefficient i ,
- -

\n- • all positive integer *n*:\n
	\n- • All polynomial functions of *U_n* are injective (simple induction)
	\n- • We can solve in polynomial time *P*(*X*) = *B* over *U_n*.
	\n- • Taking a coefficient *i*,
	\n- • Compute
	$$
	x = \sqrt[i]{\frac{\min(B)}{\min(A_i)}},
	$$
	\n- • Add to *Sol* the tree $\frac{\min(B - P(Sol))}{\min(A_i)x^{i-1}}$ until *P*(*Sol*) ∉ *B*,
	\n- • A. Return *P*(*Sol*) = *B*.
	\n\n
\n

- **1.** We can solve in polynomial time $P(U(X)) = U(B)$ over Unroll.
2. All polynomial functions over Unroll are injective. **2.** Consequences
2. All polynomial functions over Unroll are injective.
2. All polynomial functions over Unroll are injective.
-

2. Polynomials over FDDS and algorithms

Step one: polynomials over permutations
 n, m be three positive integer with $n \leq k$ and $X = X_1 + ... + X_k$ be

mutation such that *length*(X.) \leq *length*(X., .) and P(X) = **Step one: polynomials over permutations**
Let k, n, m be three positive integer with $n \le k$ and $X = X_1 + ... + X_k$ be
an permutation such that $length(X_i) \le length(X_{i+1})$ and $P(X) = \sum_{i=1}^{m} A_i X^i$ be a polynomial over permutation without c **Step one: polynomials over permutation**
Let k, n, m be three positive integer with $n \le k$ and $X = X_1 + ...$
an permutation such that $length(X_i) \le length(X_{i+1})$ and $P(X)$
 $\sum_{i=1}^{m} A_i X^i$ be a polynomial over permutation without consta **ep one: polynomials over permutations**
be three positive integer with $n \le k$ and $X = X_1 + ... + X_k$ be
ation such that $length(X_i) \le length(X_{i+1})$ and $P(X) =$
be a polynomial over permutation without constant term and
st one cancellable co

Step one: polynomials over permutat

Let k, n, m be three positive integer with $n \le k$ and $X = X_1$ +

an permutation such that $length(X_i) \le length(X_{i+1})$ and $P(\sum_{i=1}^{m} A_i X^i$ be a polynomial over permutation without constar

wit **Step one: polynomials over permutations**

Let k, n, m be three positive integer with $n \le k$ and $X = X_1 + ... + X_k$ be

an permutation such that $length(X_i) \le length(X_{i+1})$ and $P(X) = \sum_{i=1}^{m} A_i X^i$ be a polynomial over permutation without **Step one: polynomials ove**
Let *k*, *n*, *m* be three positive integer with $n \le$
an permutation such that $length(X_i) \leq len_{\mathcal{E}}$
 $\sum_{i=1}^{m} A_i X^i$ be a polynomial over permutatior
with at least one cancellable coefficient.
L

Step one: polynomials over permutations
 $n g th(AA') = lcm(len g th(A), length(A'))$ for A, A' two connected Step one: polynomials over permutations
1. $length(AA') = lcm(length(A), length(A'))$ for A, A' two connected

- **Step one: polynomials over permutations**
1. *length*(AA') = *lcm*(*length*(A), *length*(A')) for A , A' two connected
FDDS.
2. $a = lcm(a, 1) = lcm(a, a) \le lcm(a, m)$ for all integers a , m greater FDDS.
- 2. $a = lcm(a, 1) = lcm(a, a) \leq lcm(a, m)$ for all integers a, m greater than 1 for all connected component of \mathcal{I} and \mathcal{I} are \mathcal{I} . The set of \mathcal{I} and \mathcal{I} are \math Step one: polynomials over permutations
1. $length(A^{\prime}) = lcm(len(length(A), length(A^{\prime}))$ for A, A^{\prime} two conner
FDDS.
2. $a = lcm(a, 1) = lcm(a, a) \le lcm(a, m)$ for all integers a, m gr
than 1

- **Step one: polynomials over permutations**
1. *length*(AA') = *lcm*(*length*(A), *length*(A')) for A , A' two connected
FDDS.
2. $a = lcm(a, 1) = lcm(a, a) \le lcm(a, m)$ for all integers a , m greater FDDS.
- 2. $a = lcm(a, 1) = lcm(a, a) \leq lcm(a, m)$ for all integers a, m greater than $1 \Rightarrow length(X_n) \le length(C)$ for all connected component C of $P(X) - P(\sum_{i=1}^{n-1} X_i)$. Step one: polynomials over permutations

1. *length*(*AA'*) = *lcm*(*length*(*A*), *length*(*A'*)) for *A*, *A'* two conner

FDDS.

2. $a = lcm(a, 1) = lcm(a, a) \le lcm(a, m)$ for all integers *a*, *m* gr

than $1 \Rightarrow length(X_n) \le length(C)$ for all conn

- **Step one: polynomials over permutations**
1. *length*(AA') = *lcm*(*length*(A), *length*(A')) for A , A' two connected
FDDS.
2. $a = lcm(a, 1) = lcm(a, a) \le lcm(a, m)$ for all integers a , m greater FDDS.
- 2. $a = lcm(a, 1) = lcm(a, a) \leq lcm(a, m)$ for all integers a, m greater than $1 \Rightarrow length(X_n) \le length(C)$ for all connected component C of $P(X) - P(\sum_{i=1}^{n-1} X_i)$. Step one: polynomials over permutations

1. *length*(*AA'*) = *lcm*(*length*(*A*), *length*(*A'*)) for *A*, *A'* two conne

FDDS.

2. $a = lcm(a, 1) = lcm(a, a) \le lcm(a, m)$ for all integers *a*, *m* gr

than $1 \Rightarrow length(X_n) \le length(C)$ for all conne
-

- **Step one: polynomials over permutations**
1. *length*(AA') = *lcm*(*length*(A), *length*(A')) for A , A' two connected
FDDS.
2. $a = lcm(a, 1) = lcm(a, a) \le lcm(a, m)$ for all integers a , m greater FDDS.
- 2. $a = lcm(a, 1) = lcm(a, a) \leq lcm(a, m)$ for all integers a, m greater than $1 \Rightarrow length(X_n) \le length(C)$ for all connected component C of $P(X) - P(\sum_{i=1}^{n-1} X_i)$. Step one: polynomials over permutations

1. *length*(*AA'*) = *lcm*(*length*(*A*), *length*(*A'*)) for *A*, *A'* two conne

FDDS.

2. $a = lcm(a, 1) = lcm(a, a) \le lcm(a, m)$ for all integers *a*, *m* gr

than $1 \Rightarrow length(X_n) \le length(C)$ for all conne
-

Step one: polynomials over permutations
-C₂) $X + (C_4 + C_1)X^2 = 4 C_2 + 6 C_3 + 42 C_4 + 4 C_6 + 18 C_{12}$

Step one: polynomials over permutations
 $(C_3+C_2) X + (C_4 + C_1)X^2 = 4(C_2) + 6 C_3 + 42 C_4 + 4 C_6 + 18 C_{12}$

Step one: polynomials over permutations
-C₂) $X + (C_4 + C_1)X^2 = 4 C_2 + 6 C_3 + 42 C_4 + 4 C_6 + 18 C_{12}$

 $Sol = C_2$

Step one: polynomials over permutations
 $P(X) - P(C_2) = 6C_3 + 28 C_4 + 3 C_6 + 18 C_{12}$

 $Sol = C_2$

Step one: polynomials over permutations
 $P(X) - P(C_2) = 6 C_3 + 28 C_4 + 3 C_6 + 18 C_{12}$

 $Sol = C_2 + C_3$

Step one: polynomials over permutations
 $P(X) - P(C_2 + C_3) = 20(\widehat{C_4}) + 11 C_{12}$

$$
P(X) - P(C_2 + C_3) = 20C_4 + 11C_{12}
$$

 $Sol = C_2 + C_3$

Step one: polynomials over permutations
 $P(X) - P(C_2 + C_3) = 20 C_4 + 11 C_{12}$

 $Sol = C_2 + C_3 + C_4$

Step one: polynomials over permutations
 $P(X) - P(C_2 + C_3) = 20 C_4 + 11 C_{12}$

$$
P(X) - P(C_2 + C_3) = 20 C_4 + 11 C_1
$$

$$
Sol = C_2 + C_3 + C_4
$$

$$
P(X) - P(C_2 + C_3 + C_4) = 0
$$

Step one: polynomials over permutations

Step one: polynomials over permutations
Let *P* be a polynomial over permutations. If at least one non-
constant coefficient is cancellable then the polynomial is
injective and we can solve $P(X) = B$ in polynomial time. **Step one: polynomials over permutations**
Let *P* be a polynomial over permutations. If at least one non-
constant coefficient is cancellable then the polynomial is
injective and we can solve $P(X) = B$ in polynomial time. **Step one: polynomials over permutations**
Let *P* be a polynomial over permutations. If at least one non-
constant coefficient is cancellable then the polynomial is
injective and we can solve $P(X) = B$ in polynomial time.

Step two: root
 $(A, B \text{ be two connected FDDS such that } B) = b.$ **Step two: root**
Let a, b be two integers and A, B be two connected FDDS such that
 $length(A) = a$ and $length(B) = b$.
Let $t_A = min(U(A))$ and $t_B = min(U(B))$. We define the orders: **a**
 a Equipmentance CHO . Let a, b be two integers and A, B be two connected FDDS such that
 $length(A) = a$ and $length(B) = b$.

Let $t_A = \min(U(A))$ and $t_B = \min(U(B))$. We define the orders:

• $A \leq_c B$ if and only if $a < b$ or $(a = b$ and $t_A \$

• $A \leq_c B$ if and only if $a < b$ or $(a = b$ and $t_A \leq t_B)$

Step two: root
 $(A, B \text{ be two connected FDDS such that } B) = b.$ **Step two: root**
Let a, b be two integers and A, B be two connected FDDS such that
 $length(A) = a$ and $length(B) = b$.
Let $t_A = min(U(A))$ and $t_B = min(U(B))$. We define the orders: **a**
 a Equipmentance CHO . Let a, b be two integers and A, B be two connected FDDS such that
 $length(A) = a$ and $length(B) = b$.

Let $t_A = \min(U(A))$ and $t_B = \min(U(B))$. We define the orders:

• $A \leq_c B$ if and only if $a < b$ or $(a = b$ and $t_A \$

• $A \leq_c B$ if and only if $a < b$ or $(a = b$ and $t_A \leq t_B)$

Step two: root
 $(A, B \text{ be two connected FDDS such that } B) = b.$ **Step two: root**
Let a, b be two integers and A, B be two connected FDDS such that
 $length(A) = a$ and $length(B) = b$.
Let $t_A = min(U(A))$ and $t_B = min(U(B))$. We define the orders: **Step two: root**

Let a, b be two integers and A, B be two connected FDDS such that
 $length(A) = a$ and $length(B) = b$.

Let $t_A = min(U(A))$ and $t_B = min(U(B))$. We define the orders:

• $A \leq_c B$ if and only if $a < b$ or $(a = b$ and $t_A \leq t_B)$

• $A \$

- $A \leq_c B$ if and only if $a < b$ or $(a = b$ and $t_A \leq t_B)$
- $A \leq_t B$ if and only if $t_A < t_B$ or $(t_A = t_B$ and $a \leq b)$

Step two: root
 $(A, B \text{ be two connected FDDS such that } B) = b.$ **Step two: root**
Let a, b be two integers and A, B be two connected FDDS such that
 $length(A) = a$ and $length(B) = b$.
Let $t_A = min(U(A))$ and $t_B = min(U(B))$. We define the orders: **Step two: root**

Let a, b be two integers and A, B be two connected FDDS such that
 $length(A) = a$ and $length(B) = b$.

Let $t_A = min(U(A))$ and $t_B = min(U(B))$. We define the orders:

• $A \leq_c B$ if and only if $a < b$ or $(a = b$ and $t_A \leq t_B)$

• $A \$

- $A \leq_c B$ if and only if $a < b$ or $(a = b$ and $t_A \leq t_B)$
- $A \leq_t B$ if and only if $t_A < t_B$ or $(t_A = t_B$ and $a \leq b)$

Step two: root
Given an FDDS X and a positive integer p , we denote by :
 $\cdot X(p)$ the multiset of connected component of X with a cycle of lenght p . **• Step two: root**
• the multiset of connected component of X with a cycle of lenght p .
• $X(p)$ the multiset of connected component of X with a cycle of lenght p .

Step two: root
Given an FDDS X and a positive integer p , we denote by :
 $\cdot X(p)$ the multiset of connected component of X with a cycle of lenght p . **• Step two: root**
• the multiset of connected component of X with a cycle of lenght p .
• $X(p)$ the multiset of connected component of X with a cycle of lenght p .

-
- **Step two: root**
Given an FDDS X and a positive integer p , we denote by :
• $X(p)$ the multiset of connected component of X with a cycle of length p .
• $X\{p\}$ the multiset of connected component of X whose cycl **• Step two: root**

• the multiset of connected component of X with a cycle of length p .

• $X\{p\}$ the multiset of connected component of X whose cycle length is a

divisor of p . **Step two: root in the Step finds:**

an FDDS *X* and a positive integer *p*, we $X(p)$ the multiset of connected comporary $X\{p\}$ the multiset of connected comporary divisor of *p*.

-
- **Step two: root**
Given an FDDS X and a positive integer p , we denote by :
• $X(p)$ the multiset of connected component of X with a cycle of length p .
• $X\{p\}$ the multiset of connected component of X whose cycl **• Step two: root**

• $X(p)$ the multiset of connected component of X with a cycle of length p .

• $X\{p\}$ the multiset of connected component of X whose cycle length is a divisor of p . **Step two:** row **Step two:** row **an FDDS** *X* and a positive integer *p*, we $X(p)$ the multiset of connected compositivisor of *p*.

Step two: root

Let k, n, m be three positive integer with $n \le k$ and $X = X_1 + ... +$

be an FDDS such that $X_i \le c X_{i+1}$. Let B be a connected componer

min($X^m - (\sum_{i=1}^{n-1} X_i)^m$) according to $\le c$. Let $p = length(B)$. **Step two: root**

Let k, n, m be three positive integer with $n \le k$ and $X = X_1 + ... + X_k$

be an FDDS such that $X_i \le c$ X_{i+1} . Let B be a connected component of
 $\min(X^m - (\sum_{i=1}^{n-1} X_i)^m)$ according to $\le c$. Let $p = length(B)$.

T **according to . Let k**, *n*, *m* be three positive integer with $n \le k$ and $X = X_1 + ... +$
be an FDDS such that $X_i \le_c X_{i+1}$. Let *B* be a connected component
min($X^m - (\sum_{i=1}^{n-1} X_i)^m$) according to \le_c . Let $p = length(B)$.
Then **Step two: root**

Let k, n, m be three positive integer with $n \le k$ and $X = X_1 + ... + X_k$

be an FDDS such that $X_i \le_c X_{i+1}$. Let B be a connected component of
 $\min(X^m - (\sum_{i=1}^{n-1} X_i)^m)$ according to \le_c . Let $p = length(B)$.

Th

Idea of proof :

Step two: root

of **proof**:

1. The connected component *B* is the product of $m-1$

elements of $X\{p\}$ and an element of $(X^m-(\sum_{i=1}^{n-1}X_i)^{ms})(p)$ **EXEP TWO: FOOT**

Step two: **FOOT**

of proof :

1. The connected component *B* is the product of $m-1$

elements of *X*{*p*} and an element of $(X^m-(\sum_{i=1}^{n-1}X_i)^{ms})(p)$

Idea of proof :

Step two: root

of **proof**:

1. The connected component *B* is the product of $m - 1$

elements of $X\{p\}$ and an element of $(X^m - (\sum_{i=1}^{n-1} X_i)^m)(p) \Rightarrow$
 $X_n \in (X^m - (\sum_{i=1}^{n-1} X_i)^m)(p)$. **EXECT:**

Step two: root

of proof:

1. The connected component *B* is the product of $m-1$

elements of $X\{p\}$ and an element of $(X^m - (\sum_{i=1}^{n-1} X_i)^m)(p) \Rightarrow$
 $X_n \in (X^m - (\sum_{i=1}^{n-1} X_i)^m)(p)$.

Idea of proof :

- **Step two: root**

of **proof**:

1. The connected component *B* is the product of $m 1$

elements of $X\{p\}$ and an element of $(X^m (\sum_{i=1}^{n-1} X_i)^m)(p) \Rightarrow$
 $X_n \in (X^m (\sum_{i=1}^{n-1} X_i)^m)(p)$. **Elements of and an element of and an** element of $(X^m - \left(\sum_{i=1}^{n-1} X_i\right)^m)(p) \Rightarrow$
 $X_n \in (X^m - \left(\sum_{i=1}^{n-1} X_i\right)^m)(p)$.

2. The minimal unroll tree of *B* is the product of the minimal unroll tree in $X\{p\}$ raised to the roof:

The connected component *B* is the product of $m - 1$

elements of $X\{p\}$ and an element of $(X^m - (\sum_{i=1}^{n-1} X_i)^m)(p) \Rightarrow$
 $X_n \in (X^m - (\sum_{i=1}^{n-1} X_i)^m)(p)$.

The minimal unroll tree of *B* is the product of the minima roof :
The connected component *B* is the produce
dements of $X\{p\}$ and an element of $(X^m - X_n \in (X^m - (\sum_{i=1}^{n-1} X_i)^m)(p)$.
The minimal unroll tree of *B* is the product
unroll tree in $X\{p\}$ raised to the power *m* -
u
-

Step two: root
mple: square root Example: square root $\overline{2}$ $+3$ $X =$ 79

 $\begin{aligned} \mathsf{Step\ two: root} \ \end{aligned}$ is injective and we can compute it in polynomial time.

Step three: general FDDS
ree positive integer with $n \le k$ and $X = X_1 + ... + X_k$ **Step three: general FDDS**
Let k, n, m be three positive integer with $n \le k$ and $X = X_1 + ... +$
be an FDDS such that $X_i \le c$, X_{i+1} and $P = \sum_{i=1}^{m} A_i X^i$ be a polynom
without constant term and with at least one cancellab **Step three: general FDDS**
Let k, n, m be three positive integer with $n \le k$ and $X = X_1 + ... + X_k$
be an FDDS such that $X_i \le c$ X_{i+1} and $P = \sum_{i=1}^{m} A_i X^i$ be a polynomial
without constant term and with at least one cance **Step three: general FDDS**
Let k, n, m be three positive integer with $n \le k$ and $X = X_1 + ... + X_k$
be an FDDS such that $X_i \le_c X_{i+1}$ and $P = \sum_{i=1}^{m} A_i X^i$ be a polynomial
without constant term and with at least one cancella **Step three: general FDDS**

t k, n, m be three positive integer with $n \le k$ and $X = X_1 + ... + X_k$

e an FDDS such that $X_i \le c X_{i+1}$ and $P = \sum_{i=1}^{m} A_i X^i$ be a polynomial

thout constant term and with at least one cancellabl \leq_c and $p = length(B)$.
Then $length(X_n) = p$.

Step three: general FDDS
 (X_n) is the smallest cycle length in $X - \sum_{i=1}^{n-1} X_i$, we **Step three: general FDDS**
• Since $length(X_n)$ is the smallest cycle length in $X - \sum_{i=1}^{n-1} X_i$, we have that $length(X_n) \leq length(B)$. have that $length(X_n) \leq length(B)$.

- Step three: general FDDS
 (X_n) is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we **Step three: general FDDS**
• Since $length(X_n)$ is the smallest cycle length in $X - \sum_{i=1}^{n-1} X_i$, we
have that $length(X_n) \leq length(B)$.
• $length(x) = length(X_n)$, for all connected component x of $X_n{}^i$ and have that $length(X_n) \leq length(B)$.
- **Step three: general FDDS**
• Since $length(X_n)$ is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we
have that $length(X_n) \le length(B)$.
• $length(x) = length(X_n)$, for all connected component x of X_n^i and
for all integer $i \ge 1$. for all integer $i \geq 1$.

- Step three: general FDDS
(X_n) is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we **Step three: general FDDS**
• Since $length(X_n)$ is the smallest cycle length in $X - \sum_{i=1}^{n-1} X_i$, we
have that $length(X_n) \leq length(B)$.
• $length(x) = length(X_n)$, for all connected component x of $X_n{}^i$ and have that $length(X_n) \leq length(B)$.
- **Step three: general FDDS**

 Since $length(X_n)$ is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we

have that $length(X_n) \le length(B)$.

 $length(x) = length(X_n)$, for all connected component x of X_n^i and

for all integer $i \ge 1$.

 $X_n^i \in X^i$ for all integer $i \geq 1$.
- $X_n^i \in X^i$ for all integer $i \geq 1$.

- Step three: general FDDS
(X_n) is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we **Step three: general FDDS**
• Since $length(X_n)$ is the smallest cycle length in $X - \sum_{i=1}^{n-1} X_i$, we
have that $length(X_n) \leq length(B)$.
• $length(x) = length(X_n)$, for all connected component x of $X_n{}^i$ and have that $length(X_n) \leq length(B)$.
- **Step three: general FDDS**

 Since *length*(X_n) is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we

have that *length*(X_n) $\leq length(B)$.

 length(x) = *length*(X_n), for all connected component x of X_n ^{*i*} a for all integer $i \geq 1$.
- $X_n^i \in X^i$ for all integer $i \geq 1$.
- Since *length*(X_n) is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we
have that *length*(X_n) \leq *length*(B).
• *length*(x) = *length*(X_n), for all connected component x of X_n^i and
for all integer $i \$ Since $length(X_n)$ is the smallest cycle length in $X - \sum_{i=1}^{n-1} X_i$, we
have that $length(X_n) \leq length(B)$.
 $length(x) = length(X_n)$, for all connected component x of X_n^i and
for all integer $i \geq 1$.
 $X_n^i \in X^i$ for all integer $i \geq 1$.
Since P c

- Step three: general FDDS
 (X_n) is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we **Step three: general FDDS**
• Since $length(X_n)$ is the smallest cycle length in $X - \sum_{i=1}^{n-1} X_i$, we
have that $length(X_n) \leq length(B)$.
• $length(x) = length(X_n)$, for all connected component x of $X_n{}^i$ and have that $length(X_n) \leq length(B)$.
- **Step three: general FDDS**

 Since *length*(X_n) is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we

have that *length*(X_n) $\leq length(B)$.

 length(x) = *length*(X_n), for all connected component x of X_n ^{*i*} a for all integer $i \geq 1$.
- $X_n^i \in X^i$ for all integer $i \geq 1$.
- Since *length*(X_n) is the smallest cycle length in $X \sum_{i=1}^{n-1} X_i$, we
have that *length*(X_n) \leq *length*(B).
• *length*(x) = *length*(X_n), for all connected component x of X_n^i and
for all integer $i \$ Since $length(X_n)$ is the smallest cycle length in $X - \sum_{i=1}^{n-1} X_i$, we
have that $length(X_n) \leq length(B)$.
 $length(x) = length(X_n)$, for all connected component x of X_n^i and
for all integer $i \geq 1$.
 $X_n^i \in X^i$ for all integer $i \geq 1$.
Since P c .

Step three: general FDDS

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction). **contains a contains a contains a cancellable coefficient, then** P **is injective (simple induction).**
If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and contains a cancellable coefficient, then P is injec induction). **2.** Step three: general FDDS

2. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and contains a cancellable coefficient, then P is injective (simple induction). **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction). **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

- **Step three: general FDDS**

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction). **Contains a contains a contains a contains a cancellable coefficient, then** P **is injective (simple induction).**
We can solve in polynomial time the equation $P(X) = B$, by : induction). **2.** Step three: general FDDS

2. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

2. We can solve in polynomial time the equ **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

We can solve in polynomial time the equation $P(X$ **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

We can solve in polynomial time the equation $P(X$
-

- **Step three: general FDDS**

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction). **Contains a cancellable coefficient, then** P **is injective (simple induction).**
If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and contains a cancellable coefficient, then P is injective (simple induction induction). **2.** Step three: general FDDS

2. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

2. We can solve in polynomial time the equ **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

We can solve in polynomial time the equation $P(X$ **Step three: general FDDS**
 $A_i X^i$ is a polynomial without constant ter

cancellable coefficient, then P is injective

we in polynomial time the equation $P(X) =$

the length of a connected component of r

according to \le If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng
- -

- **Step three: general FDDS**

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction). **Contains a cancellable coefficient, then** P **is injective (simple induction).**
If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and contains a cancellable coefficient, then P is injective (simple induction induction). **2.** Step three: general FDDS

2. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

2. We can solve in polynomial time the equ **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

We can solve in polynomial time the equation $P(X$ **Step three: general FDDS**

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

2. We can solve in polynomial time the equati If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the
- -
	-

- **Step three: general FDDS**

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction). **Contains a cancellable coefficient, then** P **is injective (simple induction).**
If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and contains a cancellable coefficient, then P is injective (simple induction induction). **2.** Step three: general FDDS

2. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

2. We can solve in polynomial time the equ **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

We can solve in polynomial time the equation $P(X$ **Step three: general FDDS**

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

2. We can solve in polynomial time the equati If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng contains a cancellable coefficient, then *P* is injective (simple induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let *p* be the length of a connected component of min(*B* – $P(Sol)$) according to
- -
	-
	-

- **Step three: general FDDS**

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction). **Contains a cancellable coefficient, then** P **is injective (simple induction).**
If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and contains a cancellable coefficient, then P is injective (simple induction induction). **2.** Step three: general FDDS

2. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

2. We can solve in polynomial time the equ **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

We can solve in polynomial time the equation $P(X$ **Step three: general FDDS**
 $A_i X^i$ is a polynomial without constant ter

cancellable coefficient, then P is injective

we in polynomial time the equation $P(X) =$

the length of a connected component of r

according to \le If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng contains a cancellable coefficient, then *P* is injectived
contains a cancellable coefficient, then *P* is injectived
induction).
We can solve in polynomial time the equation $P(X)$
1. Let *p* be the length of a connected
-
- induction).

We can solve in polynomial time the equation $P(X) = B$, by :

1. Let p be the length of a connected component of min($B P(Sol)$) according to \leq_c ,

2. Solve the equation $C_n (U(P(X) P(Sol))) = C_n (U(B)),$

3. Roll the mini
	-
	-
	-

- **Step three: general FDDS**

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction). **Contains a cancellable coefficient, then** P **is injective (simple induction).**
If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and contains a cancellable coefficient, then P is injective (simple induction induction). **2.** Step three: general FDDS

2. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

2. We can solve in polynomial time the equ **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

We can solve in polynomial time the equation $P(X$ **Step three: general FDDS**
 $A_i X^i$ is a polynomial without constant ter

cancellable coefficient, then P is injective

we in polynomial time the equation $P(X) =$

the length of a connected component of r

according to \le If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng contains a cancellable coefficient, then *P* is injectived
contains a cancellable coefficient, then *P* is injectived
induction).
We can solve in polynomial time the equation $P(X)$
1. Let *p* be the length of a connected
-
- induction).

We can solve in polynomial time the equaring the term of a connected com
 $P(Sol)$) according to \leq_c ,

2. Solve the equation $C_n \left(U(P(X) P(So)) \right)$

3. Roll the minimum tree of this solution v

4. Add the resul We can solve in polynomial time the equation $P(X) = B$, by :

1. Let p be the length of a connected component of min(B –
 $P(Sol)$) according to \leq_c ,

2. Solve the equation $C_n (U(P(X) - P(Sol))) = C_n (U(B)),$

3. Roll the minimum tree o
	-
	-
	-
	-

- **Step three: general FDDS**

1. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction). **Contains a cancellable coefficient, then** P **is injective (simple induction).**
If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and contains a cancellable coefficient, then P is injective (simple induction induction). **2.** Step three: general FDDS

2. If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

2. We can solve in polynomial time the equ **Step three: general FDDS**

If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and

contains a cancellable coefficient, then P is injective (simple

induction).

We can solve in polynomial time the equation $P(X$ **Step three: general FDDS**
 $A_i X^i$ is a polynomial without constant ter

cancellable coefficient, then P is injective

we in polynomial time the equation $P(X) =$

the length of a connected component of r

according to \le If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng If $P = \sum_{i=1}^{m} A_i X^i$ is a polynomial without constant terms and
contains a cancellable coefficient, then P is injective (simple
induction).
We can solve in polynomial time the equation $P(X) = B$, by :
1. Let p be the leng contains a cancellable coefficient, then *P* is injectived
contains a cancellable coefficient, then *P* is injectived
induction).
We can solve in polynomial time the equation $P(X)$
1. Let *p* be the length of a connected
-
- induction).

We can solve in polynomial time the

1. Let *p* be the length of a connecte
 $P(Sol)$) according to \leq_c ,

2. Solve the equation $C_n \left(U(P(X))\right)$

3. Roll the minimum tree of this solu

4. Add the resulting c We can solve in polynomial time the equation $P(X) = B$, by :

1. Let p be the length of a connected component of min(B –
 $P(Sol)$) according to \leq_c ,

2. Solve the equation $C_n (U(P(X) - P(Sol))) = C_n (U(B)),$

3. Roll the minimum tree o
	-
	-
	-
	-
	-

Thanks for your attention

References

- **References**
1. Dennunzio, A., Dorigatti, V., Formenti, E., Manzoni, L., Porreca,
A.E.: Polynomials over the semiring of dynamical systems.
2. Naquin, E., Gadouleau, M.:Factorisation in the semiring of finite
- **:ferences**
Dennunzio, A., Dorigatti, V., Formenti, E., Manzoni, L., Porreca,
A.E.: Polynomials over the semiring of dynamical systems.
Naquin, E., Gadouleau, M.:Factorisation in the semiring of finite
dynamical systems. **References**
2. Dennunzio, A., Dorigatti, V., Formenti, E., Manzoni, L., Porreca,
2. Naquin, E., Gadouleau, M.:Factorisation in the semiring of finite
4. Maquin, E., Gadouleau, M.:Factorisation in the semiring of finite
3. dynamical systems. **References**

3. Dennunzio, A., Dorigatti, V., Formenti, E., Manzoni, L., Porreca,

4.E.: Polynomials over the semiring of dynamical systems.

2. Naquin, E., Gadouleau, M.:Factorisation in the semiring of finite

dynamical
- semiring of finite deterministic dynamical systems